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MATHEMATICAL REPRESENTATION OF COMPACTION CORE HEIGHT WHEN MEDIUM CRYSTAL STRUCTURE FAILURE

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Викладено результати аналізу формування ядра ущільнення при руйнуванні середовища та визначення залежностей геометрії (висоти) ядра ущільнення від фізико-механічних властивостей атомарних структур середовища і технологічних характеристик ріжучого інструменту.

Ключові слова: ядро ущільнення, швидкість руйнування, внутрішнє джерело енергії, частота власних коливань, енергетичне джерело.

The article represents the results of compaction core formation analysis when medium failure. Also it determines compaction core geometry (height) dependencies on physical-mechanical properties of atomic structure and cutter processing characteristics.

Key words: compaction core, failure rate, internal energy source, natural oscillation frequency, energy source.

Изложены результаты анализа формирования ядра уплотнения при разрушении среды и определение зависимостей геометрии (высоты) ядра уплотнения от физико-механических свойств атомарных структур среды и технологических характеристик режущего инструмента.

Ключевые слова: ядро уплотнения, скорость разрушения, внутренний источник энергии, частота собственных колебаний, энергетический источник.

Introduction. Before failure, a compaction core forms where the tool contacts with the medium. Core volume compaction and extension, which change tool geometry, consume substantial (more than 50%) part of drive energy. At that, the stope is broken not with the tool's cutting edge, but with a compaction core. Owing to this, failure energy-output increases, process of energy-efficiency reduces.

According to the Directive of the European Parliament and the Council "on effectiveness of energy final use and energy services"; and also on Council's Directive 93/76/EEC abolition of the 5^{th} April 2006 2006/32/EC [1], energy-efficiency increase of rock-mass failure stands as a rather urgent problem.

The objective. The analysis of compaction core formation process when stope failure; determination of compaction core geometry (height) dependencies on physical-mechanical properties of medium atomic structure and cutter's processing characteristics. That demands mathematical models development, which connect compaction core height with medium atomic structure under failure. The novelty lies in switching from failure mechanics classical theory to failure model at the level of medium atomic structure in compliance with quantum physics laws.

The outline. When analyzing modern rock failure theories, H. H. Kausch [2], V. Regel [3], I. Narisava [4] and others can be defined as initiators. According to their studies, crystal lattice potential energy is calculated through atomic balance displacement, atomic interaction force and free electron energy. That gives the opportunity to switch from classical to molecular-wave failure theories. They show stope structure interaction (i.e. set of plasticity and elasticity forces) and represent continuity of physical-mechanical constants of medium under failure. The drawbacks of considered theories lie in the continuity of physical-mechanical constants, which do not accommodate to various working conditions of energy sources and medium atomic structures state transition.

In order to analyze failure, the stope is examined at the atomic level. The presence of external and internal stimuli (gravity, rheological properties of interatomic bonds, deformation of the medium under failure, force weak and strong interactions) causes oscillation of the structure atoms.

The natural oscillation frequency of a particle (an atom) ω, s^{-1}, is defined by means of Einstein model [5]:

$$\omega = \varepsilon / \hbar , \tag{1}$$

where ε – energy of elementary particle (atom), J; $\hbar = 1{,}054571726 \cdot 10^{-34}$ – Dirac constant, *J·s*.

According to the technical data, SBSh-250MN machines are meant for drilling rocks with strength factor *f* = 12–18 by the Protodiakonov scale [11]. Rocks with *f* = 12, 16, 18 (quartz, diabase, porphyry) are selected for modeling. According to their chemical composition, the main atomic structure, bonds of which are to be destructed, are distinguished. Energy value of the structure components are represented in the Table 1.

Table 1. Factors of atomic natural energy

Atom	Natural energy ε, eV	Source
Silicium (Si)	1,119	[8]
Ferrum (Fe)	2,2	[7]
Calcium (Ca)	5	[6]

For silicium (Si), the natural oscillation frequency of atomic lattice, s^{-1}:

$$\omega = 1.119 \cdot 1.6 \cdot 10^{-19} / 1.0545717 26 \cdot 10^{-34} = 1.69775080 8085 \cdot 10^{15} .$$

The results of the calculation by (1) for silicium and other atoms are represented in the Table 2.

Table 2. Natural oscillation frequency of the atoms

Atoms	The natural oscillation frequency of the atoms, ω_0, s^{-1}
Silicium (Si)	1697750808085000
Ferrum (Fe	3337847880060000
Calcium (Ca)	7586017909227000

The compaction core height when stope failure in the system "External source – medium" [9]:

$$h_я = \frac{\sqrt{N_П^2 + N_P^2} \cdot \eta_П \cdot \ln((0.00013 \cdot E)^{1.67} \cdot \pi \cdot (1 - \nu^2)/(0.152 \cdot E))}{28.2 \cdot a_k \cdot 0.0083 \cdot E \cdot \eta_P \cdot N_i \cdot \sqrt{V_П^2 + V_P^2}}, \quad (2)$$

where $N_П$ – the mean value of the feed drive capacity, *W*; N_P – the mean value of the cutting drive, W; $\eta_П$ – efficiency factor of the feed and cutting drives, r. u.; E – Young modulus, first-kind, Pa; ν – Poisson ratio, r. u.; a_k – contact width of the tool with the stope, m; η_P – efficiency factor of the reduction gear, r. u.; N_i – quantity of the cutting tools, p; $V_П$ – mean speed of the feed, m/s; V_P – mean speed of cutting, m/s.

The drawback of the suggested model is that there is no natural oscillation frequency and internal atomic energy influence on the height of the occurring compaction core. The transfer to the atomic level is accomplished according to the laws of quantum mechanics. The presence of the Planck constant is a distinct feature of using the laws of quantum mechanics. When transferring to the main factor of quantum mechanics – the Planck constant – the Dirac constant was used [10].

$$h = \hbar \cdot 2\pi, \quad (3)$$

where $h = 6{,}62606957 \cdot 10^{-34}$ – Planck constant, *J·s*.

The Ludolfian number (π) is substituted, because it describes not energy properties, but geometric ones. In this paper, the medium under failure is considered as an oscillatory contour – a system with internal energy source – of atomic medium structure. That is why the internal oscillation frequency and atomic structure energy influence on the external source's specifications is important. The Dirac constant describes these properties. The main laws of quantum mechanics are projected on the classical failure theory. Taking the Dirac constant into account, the formula (2) describes experimental new system "Energy source – system with an internal source of energy" and assumes the following form:

$$h_я = \frac{\sqrt{N_П^2 + N_P^2} \cdot \eta_П \cdot \ln((0.00013 \cdot E)^{1.67} \cdot (h/2\hbar) \cdot (1 - \nu^2)/(0.152 \cdot E))}{28.2 \cdot a_k \cdot 0.0083 \cdot E \cdot \eta_P \cdot N_i \cdot \sqrt{V_П^2 + V_P^2}}. \quad (4)$$

Taking the Einstein model (1) into account, the compaction core height in the system “Energy source – system with an internal energy source”:

$$h_я = \frac{\sqrt{N_П^2 + N_P^2} \cdot \eta_П \cdot \ln((0.00013 \cdot E)^{1.67} \cdot (h\omega / 2\varepsilon) \cdot (1-\nu^2)/(0.152 \cdot E))}{28.2 \cdot a_k \cdot 0.0083 \cdot E \cdot \eta_P \cdot N_i \cdot \sqrt{V_П^2 + V_P^2}}. \quad (5)$$

In order to conduct mathematical model analysis, the specifications of such rock-drilling machines as SBSh-250MN-32, SBSh-250MN-32D, SBSh-250MN-32KP-18, SBSh-160/200-40, SBSh-160/200-40D (from the PTC “Rudgormash” catalogues [11]) were used. They are represented in the Table 3.

Table 3. SBSh rock-drilling machines specifications

The model of a rock-drilling machine	SBSh-250MNA-32	SBSh-250 MNA-32D	SBSh-160/ 200-40D
Rock strength factor by the scale of Protodiakonov f	f = 6–20	f = 6–20	f = 6–18
Borehole diameter a_k, mm	160, 170, 190, 215, 250, 270	170–311	160, 171, 215
Weight capacity upper limit $F_П$, kN	294	340	235
Drilling assembly rotation frequency upper limit ω_p, min^{-1}	120	150	140
Maximum torsional moment M, N·m	17400	13000	7000
Drilling assembly speed of descent/ ascent $V_П$, m/min	15/15	20/20	15/15

Linear cutting speed, [12]:

$$V_p = \omega_p \cdot (a_k/2). \quad (6)$$

Feed drive capacity [13]:

$$N_П = F_П \cdot V_П. \quad (7)$$

Cutting drive capacity [14]:

$$N_P = M \cdot \omega_p. \quad (8)$$

Substituting (6)–(8) for (5), compaction core height:

$$h_я = \frac{\sqrt{(F_П \cdot V_П)^2 + (M \cdot \omega)^2} \cdot \eta_П \cdot \ln((0.00013 \cdot E)^{1.67} \cdot (h\omega / 2\varepsilon) \cdot (1-\nu^2)/(0.152 \cdot E))}{28.2 \cdot a_k \cdot 0.0083 \cdot E \cdot \eta_P \cdot N_i \cdot \sqrt{V_П^2 + (\omega_p \cdot a_k/2)^2}}. \quad (9)$$

For the first time the mathematical model is introduced, which differs from the existing ones as it takes rocks' physical-mechanical properties considering medium atomic structure into account. It also defines their dependence on changes in specifications of technical data in different models of existing rock-drilling machines. In order to analyze the suggested model, all necessary data are represented in the Tables 4 and 5.

Table 4. Atoms' physical properties

Atoms	Poisson ratio	Source	Young modulus, GPa	Source
Silicium (Si)	0.266	[15]	109	[18]
Ferrum (Fe)	0,28–0.29	[16]	190–210	[19]
Calcium (Ca)	–0,27	[17]	26	[20]

Table 5. Specifications of SBSh rock-drilling machines' drives

Efficiency factor of the cutting drive, r.u.	Source	Efficiency factor of the feed drive, r. u.	Source	Efficiency factor of the reduction gear, r. u.	Source
0,65	[22]	0,7	[22]	0,95	[23, 24]

When SBSh-250MN-32 is working with the stope with silicium (Si) atomic structure, the height of the compaction core, m:

$$h_{я11} = \frac{\sqrt{(294000 \cdot 15/60)^2 + (17400 \cdot 120/60)^2} \cdot (0{,}65 \cdot 0{,}7) \cdot \ln((0.00013 \cdot 109 \cdot 10^9)^{1.67}}{28.2 \cdot 0{,}160 \cdot 0.0083 \cdot 109 \cdot 10^9} \times$$

$$\times \frac{(6{,}62606957 \cdot 10^{-34} \cdot 1697750808\,085000 / 21{,}054571726 \cdot 10^{-34} \cdot 1{,}119 \cdot 1{,}6 \cdot 10^{-19}) \cdot}{0{,}95 \cdot 1 \cdot} \times$$

$$\times \frac{(1-0{,}266^2)/(0.152 \cdot 109 \cdot 10^9))}{\sqrt{(15/60)^2 + ((120/60) \cdot (0{,}160/2))^2}} = 1.62 \cdot 10^{-4}.$$

The core height of the above-mentioned model for the size spectrum of rock-drilling machines when interacting with different atomic structures is calculated by analogy. The comparison of compaction core height values when SBSh-250MN is working with rocks of different atomic structures and with changes in borehole diameter and cutting speed is represented in the Fig. 1 (*a*–*c*).

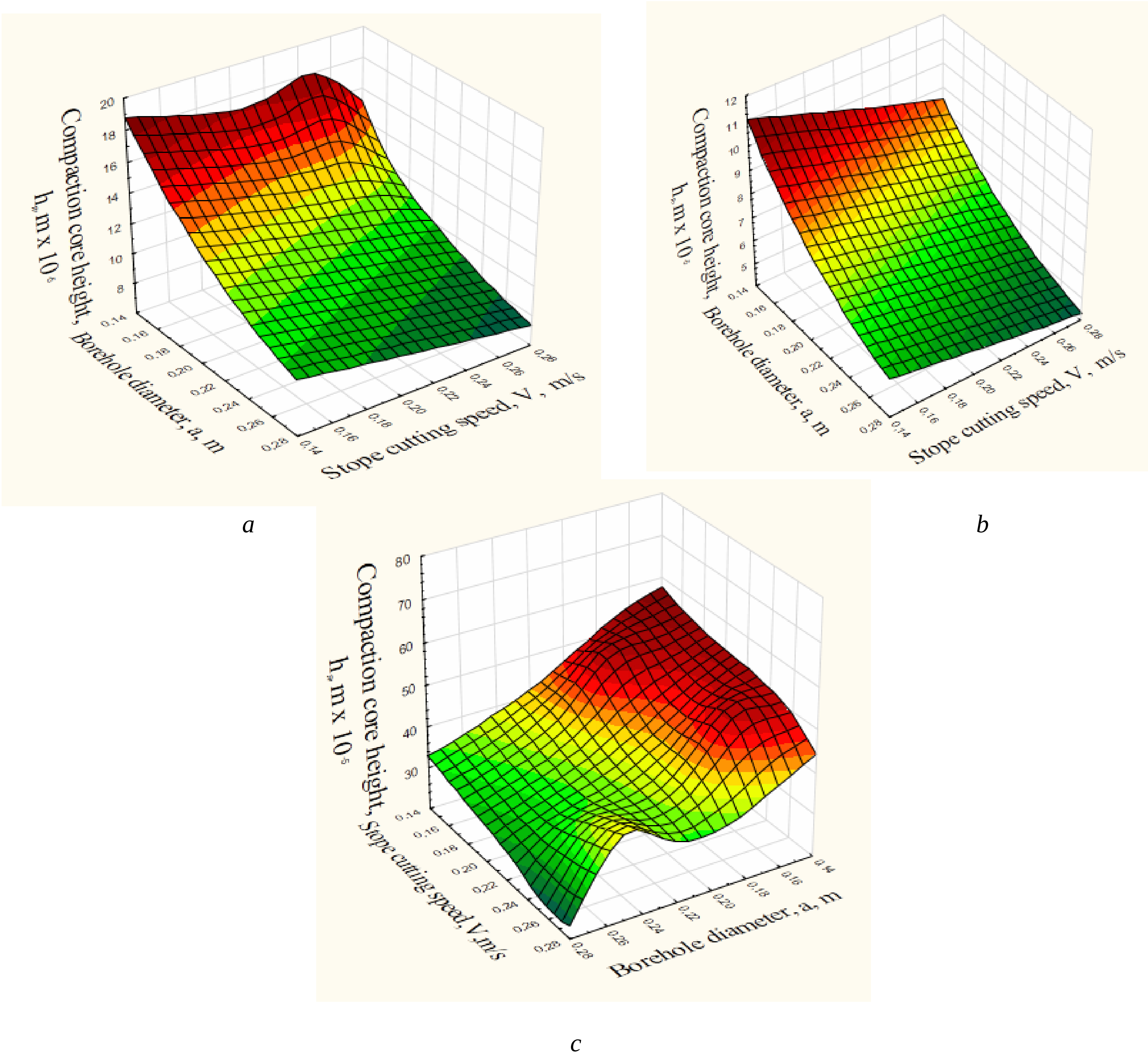
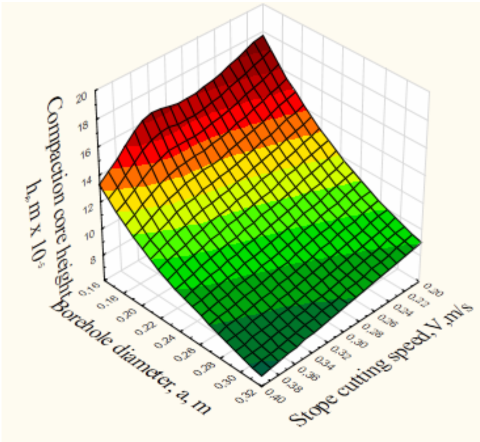
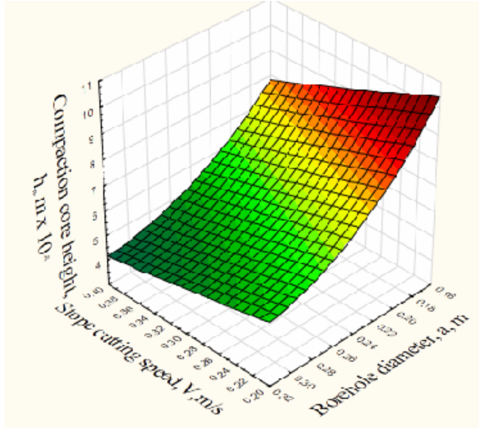


Fig. 1. Graphs of compaction core height dependencies on cutting speed and borehole diameter of SBSh-250MNA-32: *a* – silicium atomic structure, *b* – ferrum, *c* – calcium

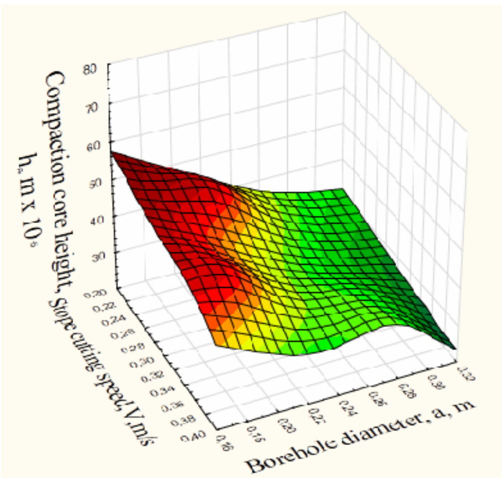
The comparison of the compaction core height when SBSh-250MN-32D and SBSh-160/200-40D are working with the rocks of different atomic structure, changes in cutting speed and borehole diameter is represented in the Fig. 2.


Compaction core height, h, m x 10
Borehole diameter, a, m
Stope cutting speed, V, m/s

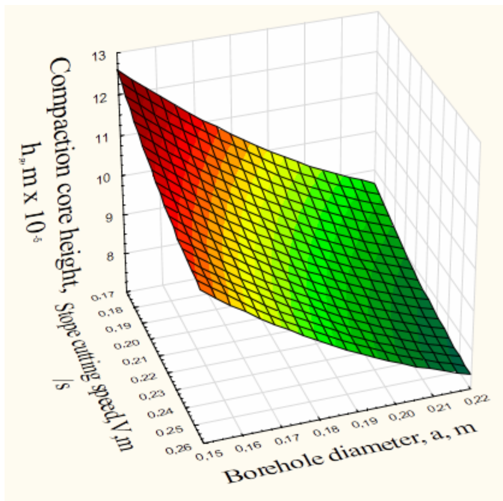
a


Compaction core height, h, m x 10
Stope cutting speed, V, m/s
Borehole diameter, a, m

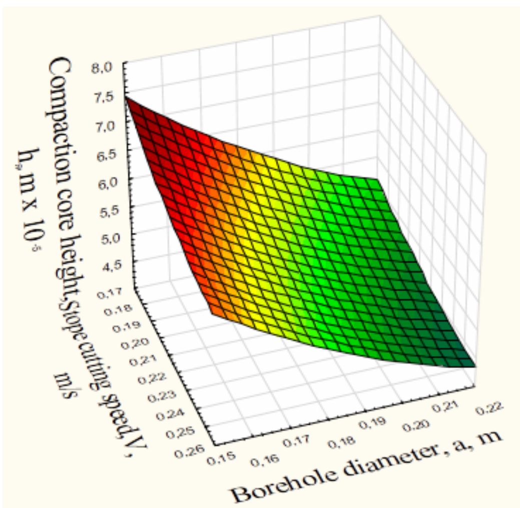
b


Compaction core height, h, m x 10
Stope cutting speed, V, m/s
Borehole diameter, a, m

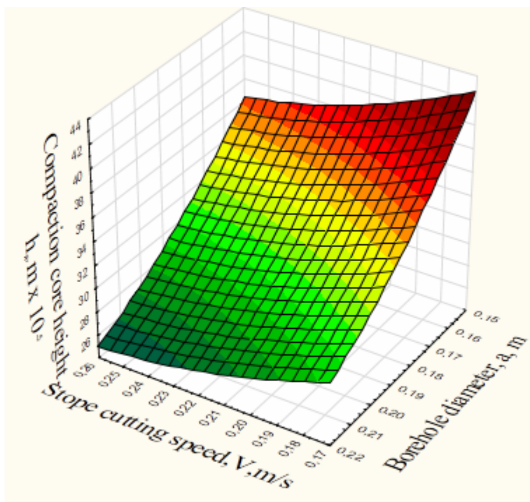
c


Compaction core height, h, m x 10
Stope cutting speed, V, m/s
Borehole diameter, a, m

d


Compaction core height, h, m x 10
Stope cutting speed, V, m/s
Borehole diameter, a, m

e


Compaction core height, h, m x 10
Stope cutting speed, V, m/s
Borehole diameter, a, m

f

Fig. 2. Graphs of compaction core height dependencies on cutting speed and borehole diameter projection: *a–c*: SBSh-250MN-32D; *d–f:* SBSh-160/200-40D; *a, d* – silicium; *b, e* – ferrum; *c, f* – calcium

When the speed increased by 0,1 m/s, the compaction core height reduced by 16 mcm. When the borehole diameter increased by 55 mm, the compaction core height reduced by 27 mcm. Based on the dependency plane character, Fig. 1, the borehole diameter increase is more preferable than the cutting speed increase. To reduce the core height by 25 mcm when working with SBSh-250MN, it is enough to increase the borehole diameter by 60 mm (when ferrum prevails in the medium) or to increase the cutting speed by 0,14 m/s, twice as much that is. The compaction core height of silicium, 96 mcm, is up to 4 times smaller than the compaction core height of calcium atomic structure, 325 mcm. The calcium is considered to be an auxetic (it has negative Poisson ratio) [17]. That is, it has the property to increase its volume under compression. Therefore, the compaction core height of the rocks that mainly contain calcium will be bigger. The compaction core height of ferrum atomic structure, 56 mcm, is smaller than of silicium one due to bigger elastic modulus. The smallest compaction core, 42,6 mcm, occurs when SBSh-250MN-32D is working with the stopes that mainly contain ferrum, maximum borehole is 0,311 m and maximum cutting speed is 0,38 m/s. Ferrum has elastic properties, namely Young modulus, 190–210 GPa, which defines bigger material fragility than silicium and calcium. The compaction core heights when working with SBSh-250MN-32D are smaller, because bigger diameters of the cutting tools are used – up to 311 mm.

Conclusions and recommended practices

1. For the first time the mathematical model is introduced, which is adapted to analysis of compaction core formation and differs from the existing ones as it takes rocks' changeable physical-mechanical properties of the stopes and their atomic structure into account. It also defines their dependence on changes in specifications of technical data in different models of existing rock-drilling machines (cutting tool diameter and cutting speed).

2. When the speed increased by 0,1 m/s, the compaction core height reduced by 16 mcm. When the borehole diameter increased by 55 mm, the compaction core height reduced by 27 mcm.

3. The compaction core height of silicium, 96 mcm, is up to 4 times smaller than the compaction core height of calcium atomic structure, 325 mcm. The calcium is considered to be an auxetic (it has negative Poisson ratio) [17]. The compaction core height of the rocks that mainly contain calcium is bigger. The compaction core height of ferrum atomic structure, 56 mcm, is smaller than of silicium one due to bigger elastic modulus.

4. The compaction core heights when working with SBSh-250MN-32D are smaller, because bigger diameters of the cutting tools are used – up to 311 mm.

The smallest compaction core, 42,6 mcm, occurs when SBSh-250MN-32D is working with the stopes that mainly contain ferrum, maximum borehole is 0,311 m and maximum cutting speed is 0,38 m/s.

5. The borehole diameter increase is more preferable than the cutting speed increase; because compaction core height plane has softer character when the borehole diameter is increased (plane slope angle is twice as bigger). To reduce the core height by 25 mcm when working with SBSh-250MN, it is enough to increase the borehole diameter by 60 mm (when ferrum prevails in the medium) or to increase the cutting speed by 0,14 m/s, twice as much that is.

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